SUSTAINABLE DEVELOPMENT MODELING (VANU-OPREAN-BUCUR MODEL)

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ABSTRACT: The concept of sustainability relies heavily on the idea that human activities are dependent on the environment as well as resources. Health, social security and economic stability of society are sine-qua-non elements of life. Now, more than ever, it is essential to initiate dialogue and further identify pertinent measures for the future. We believe that it is imperative to become aware that society should be on its way from a consumer-based one to a sustainable society. We have noticed that, in spite of much talk about socio-economic restructuring and the necessity of improving the quality of life and human performance, little has been actually done in this respect, mainly due to the lack of interaction among those who are aware of the conceptual limitations to the current paradigm and those who are actually able to operate coherently and fluently with the series of consequences entailed by the new paradigm of sustainability. These are the reasons and the premises that have determined us to develop in the present paper a mathematical model of sustainable development, in order to strengthen the scientific approach to this concept. It is well known that mathematical modelling provides useful instruments for the connections among economic, social and environmental data, as well as predictions of the impact of environmental pollution, it makes available action policies, it helps anticipate pollution peaks and estimate the amount of non-renewable resources. Such mathematical models help create data bases that may be processed by means of performative software thus engendering pertinent and relevant conclusions regarding decision-making processes.

The mathematical model set forth in the present paper is meant to bring forth the sustainability paradigm instead of the optimality paradigm. This is a functional model that involves the analysis of a dynamic system whose performance may be assessed by means of specific methods with a high degree of generalization. The present paper is meant to highlight the relevance of sustainable development to modern man`s life, an individual acting in keeping with a high sense of morality and decency, as well as an individual in charge of making decisions at the economic, political, governmental or private level. Therefore, we consider that sustainability represents a goal for those who understand the globalization-induced crisis and assume commitment to the development of the post-globalization society, i.e. a knowledge-based society.

1. INTRODUCTORY REMARKS

Sustainability should represent more than a mere project undertaken by various specialists in view of intelligent resource management as well as more than a simple concept, widely used in certain contexts. This is a paradigm where the future is designed as a balance of its three components (economic, social and environmental) in view of developing and improving life quality. Sustainable development is a rather hard-to-define concept, since it continuously changes and enriches with multiples meanings.

Sustainable development should be undertaken by each individual an it represents a process of continuous and permanent improvement, of learning and training. We should all make a lasting contribution to the various aspects of our existence and for the following generations, as well. This is the only solution to ensuring the future of mankind.

Industrial organizations are engaged in an attempt to increasingly develop a responsible behavior aiming to observe the principles of the „sustainable development" concept, to integrate the policy and principles of sustainable development in their activity and business in order to become more competitive compared to other organizations. An analysis and control of the various risk categories that might affect such an organization shall be performed from an economic, social and environmental perspective.

Any industrial organization operating in keeping with the principles of sustainable development should set forth a long-term perspective and approach, without overlooking, though, the opportunities that may come up in the meantime. This might endorse the creation of sustainable value and long-term benefits that help support both business and community development in a sustainable manner.

The major objective of the present paper is to design a valuable instrument – a mathematical model for the sustainable development of industrial organizations. This enterprise entails new elements necessary for the fundamenting, assessment, drawing up and coordination of the sustainable development strategy of an industrial organization; moreover, it also helps survey the accomplishments and also compare the results of sustainable development.

The model we have set forth cannot be a rigid or static project, on the contrary, it need be a flexible, dynamic and adjustable model in keeping with new factors. Undoubtedly, unforeseen and unpredictable elements cannot be ignored since they represent a part of the world with a potential to influence the developmental trend of society and its organizations. No researcher can anticipate the events that are going to happen in time, or the uncontrollable elements that might occur in the indicator identification. There is no recipe including the ingredients for the meaning of life, the sustainable development. No researcher can set forth a rigid, static recipe for the governance of the world or organizations in the future.

Sustainable development is directly and vitally connected to some indicators (organizational wellness, occupational health, innovation, ethics). Sustainable development cannot function without these indicators, and it shall remain only an intention, an ideal.

Mention should be made of the fact that sustainable development of industrial organizations, and not only, can only be achieved by a strong, genuine cooperation among specialists from all fields of activity: economists, sociologists, environmental specialists, engineers, mathematicians, psychologists, physicians, etc. Society must become aware that we need to pool our efforts in order to achieve a common goal – sustainable development. Interdisciplinary studies are the optimum solution for the approach to and results of sustainable development.

Sustainability and innovation should benefit from a threefold support, aforementioned, by means of modern technical methods, state-of-the-art technology, undergoing a process of continuous development.

The technique of mathematical modelling proves to be useful in all scientific fields. It is well known that a qualitative description of any phenomenon subject to study as well as the expression of certain laws are not enough unless they are associated with a thorough analysis of the quantitative laws governing the respective phenomenon. Therefore, we wish to measure whatever can b measured as well as to make measurable whatever has not been measured yet.

A mathematical model [5] can be defined as the mathematical problem associated with any topic undergoing analysis. This model is not necessarily the most accurate or the most appropriate.

The study of dynamic systems, for instance, shows that the fundamental component of a mathematical model of any dynamic process (economic, educational, social, biological, chemical, etc) represents a set of equations connecting the variables and parameters that describe the state of the particular system or a purpose function, on the variables subject to limitations (restrictions).

In the process of selecting variables the researcher will always make a compromise between the complexity of the system and the manner of objective representation of the features considered essential to its analyses. The role of the parameters is to represent the control mechanism of the process as well as to adjust the model curve for experimental data.

In a dynamic process, the time variable, t, is always included, implicitly or explicitly. In this context, we can distinguish:

- short-term or long-term behaviour models;
- continuous or discreet models;
- the lack or presence of delayed effects in certain models.

Mathematical models are the result of an interdisciplinary study, performed continuously and consistently. There is no general method of drawing up a mathematical model.

The present mathematical model aims to generate an innovation model that might lead to hierarchy and which can also justify the choice of the indicators defining the innovative process.

Specific case. Application of mathematical models to modelling the components of sustainable development

The problematics of measuring the economic and environmental indicators occurring in the components of sustainable development has been subject to heated debate in specialized literature, the measurement methods are well fundamented and known. Therefore, we are not going to focus on them any longer, but rather on the measurement of social indicators and social phenomena modelling.

The matter of quantitative analysis of the social component of sustainable development for any industrial organization represents, to a great extent, a matter its scientific knowledge in correlation with different types of relations between the employee and the industrial organization (appreciative, emotional, intuitive, tradition-based).

However, at any given moment and for any specific field, in relation to the scientific aspect of evaluating the social component and given a limited level of development for investigation methods, there will be relative limits to the efficiency of quantitative analysis, and such limitations are going to increase ceaselessly once the investigation methods and techniques improve. The application of computational calculus will open up new possibilities not only for quantitative analysis (measurement), but also for the social component modelling. Among other things, we can refer to the potential computer-aided modelling of decision-making processes, conflicting situations, etc.

The fundamental characteristic of social indicators is the fact that the object of knowledge is the subject, i.e. the conscience bearer.
An essential characteristic of the social component is its stochastic nature, as a consequence of the fact that we have to deal with various actions of multitudes of people, the existence of a great number of qualitative indicators which cannot b easily subject to a quantitative description. The social component is characterized by complexity and a great multitude of factors, subject to the influence of a significant number of objective and subjective conditions.
Since the main difficulties that come up in the development of the technology of sociological measurement is connected to the measurement of qualitative characteristics, therefore the matter of social component measurement is rightly associated to the problem of measuring qualitative characteristics and their representation in a quantitative form. Consequently, throughout solving this problem, the researcher should not merely confine to identifying qualitative indicators within the phenomenon subject to study, but rather highlight the extent of the presence of such characteristics, that is to measure them.
For instance, researching the attitude to work, organizational wellness, the scientist should find or design the indicators that enable him to express them in a quantitative manner.
Therefore, the main problem of mathematical modelling is the appropriate choice of qualitative indicators and their processing by means of non-parametric statistics (emphasizing associations), drawing up a more complicated apparatus for social statistics that considers the specificity of social phenomena, social component of the sustainable development function, improvement of correlational analysis and factorial analysis.
The authors of the present paper consider that the researcher should possess accurate and thorough knowledge of the socio-economic and environmental characteristics of the area where the social process subject to study is operational, or of the industrial organization whose sustainable development the researcher intends to assess. In the absence of such information, all mathematical models and accurate methodology for the study of specific phenomena will remain incomplete.
How stable, though, are the social indicators and their impact on the sustainable development of an industrial organization? Here is a question to which the present paper has provided an answer which is going to determine the perspectives and limitations of measuring *f2* social component and social-psychological ones.
We believe that the accurate research means not only a quantitative expression of basic variables (i.e. measurement) but also a standardized description, subject to a single interpretation (i.e. mathematical) of the causal-functional connections among the elements of the social component undergoing research.
Social component modelling requires the measurement of various social indicators, both qualitative and quantitative, identifying the relations among them and summing up the information they carry. In a more general sense, *f2* social component modelling as well as the other two components of the sustainable development of an industrial organization, means the reflection of reality as determined sets of symbols, signs, judgments, etc.
The most important problem of modelling was to make existing qualitative representations about social phenomena official. The structure of an industrial organization, headcount, personality structure, human interaction, social norms, etc. all these may be theoretically described by means of a formal language. This process of designing a „scientific language” will do away with a number of ambiguous notions still present in contemporary literature, will help clarify the interpretation of experience and experiment, will create a basis for further deepening of core aspects of the sustainable development research. The following significant and „active” issue is to provide proper description of various industrial organizations, i.e. a hierarchy, and a comparative approach includes the social component values of the sustainable development function.
Mention should be made of the “optimal theory” of all various ways of using the social component. Its essence resides in the fact that the multitude of potential states of the system, industrial organizations, is considered to be regulated by means of a certain function – the objective function of the system – and consequently, it might determine its future states by solving appropriate problems of maximum and minimum. Some of the special mathematical disciplines employed in this context are: management theory, usefulness theory, mathematical programming, etc.
In order to identify the optimal relations of the costs related to professional training, health protection, social protection, it is imperative to find a thorough mathematical expression of the quality indicators for the respective processes and to establish the limitations of their potential mutual influence.

The process of mathematical transformation of the social component requires the solving of two additional problems and simultaneously different: identification of correlation dependencies, for building up multifactorial models and researching the optimal criteria of employee activity in an industrial organization.

Mention should also be made that the study of social indicators by means of mathematical models is currently confronted with some obstacles that might be accounted by: insufficient development of sociological research, limited scale, restricted problematics; poor theoretical perspectives of many sociological problems; lack of necessary coordination of the efforts of sociologists, economists, demographers, psychologists, mathematicians, cyberneticians, logicians etc; poor knowledge by sociologists of the mathematical apparatus; low level of development in the mathematical fields of the specific "social" fields of application; insufficient knowledge of the calculus technique specially adapted to solving sociological research problems.

2. HYPOTHESES AND ORIGINAL METHODOLOGY

Hypotheses

Such a model has the following components:

- the objective function, purpose function, DDOI function;
- variables (indicators, parameters) and their influence factors;
- limitations applied to variables.

The function of sustainable development in industrial organizations may be related to the following aspects:

- it represents a global indicator of sustainable development in industrial organizations, dependent on partial indicators (parameters);
- it provides a method for the assessment of sustainable development in industrial organizations, for an estimation of their chronological evolution, or optimal accomplishment;
- the possibility of making various predictions, forecasts regarding sustainable development, on the basis of its values;
- performing comparative approaches to the functions of sustainable development of several organizations, and the conclusions refer not only to their „increase" but also to a „sensible" (in terms of ethics, morality) increase of sustainable development;
- it provides a mathematical solution to various optimization problems – at the economic, social, environment levels - of the purpose functions, that might occur in the expression of the DDOI function by employing operational research methods. The methods specific to mathematical analysis, differential calculus cannot be applied, since the definition fields do not meet the necessary requirements (structure). Differential calculus may be used for the study of indicator optimization, as these indicators may be considered both as time functions and interdependent by functional relations.
- fDDOI analysis may be approached by means of numerical methods referring to calculating certain functions by drawing up discreet value tables. Knowing the values of an f function at some points of the definition interval, called interpolary nodes, any other function sharing similar values with f in these nodes, will be an interpolary function for f. For instance, in the case of linear interpolation, the f graph is represented as a polygonal line where the end/top is *(xi, f(xi))*. Other interpolary functions: Lagrange, Legendre, Hermite, Cebîșev polinoms.
- there is a minimum-maximum mathematical model, where production (x_{20}) , represents the maximum function, and the secondary effects entailed by technological activities $(x_{n+20}, \dots, x_{n+22}, x_{n+m+18}, x_{n+m+20}, x_{n+m+21}, etc.)$ represent the minimum functions;
- the mathematical model is applicable to optimization criteria (criteria used for profit maximization, maximization of renewable resources consumption, minimization of production costs and energy costs) or it can be approached by means of multifactorial optimization (also called Pareto optimization). The general expression of the model (problem) for linear programming is [4]:

$$\begin{cases} min\ (max) \sum_{j=1}^{n} c_j x_j \\ \sum_{j=1}^{n} a_{ij} x_j \leq (\geq) b_i, i = \overline{1,m}, \\ x_j \geq 0,\ \ j = \overline{1,m} \end{cases} \tag{1}$$

- Operational research methods may be: deterministic (linear programming, critical way), probabilistic (waiting threads, Pert) and simulative (Monte Carlo, dynamic simulation).

The orientation of sustainable development analysis towards optimal solutions to problems is also facilitated by marginal calculus thus enabling the identification of maximum or minimum circumstances.

The essence of marginal calculus and its usefulness in the analysis start from the functional relations (9), (10), that is: $Y = f(X), Y = f'(X), Y = f_1(X), Y = f_2(X), Y = f_3(X)$.

It is well known that the graphic representation of any function may be accomplished by means of points, histograms, broken or curved lines, depending on the type of operation: discreet (finite) variations or continuous (infinitesimal) operations.

Considering the very small variations of the X variable, we might consider that the Y function is represented by means of smooth and continuous curved lines.

Under these circumstances, the Y marginal value expresses the variation (±) of that function, as a response to X variation. Therefore, we may consider marginal value as an absolute elasticity of the function that expresses its sensitivity to the value of the argument, calculated as a ratio of the increases ΔY and ΔX.

Between the X and Y variables of the function, the following values are defined and calculated:

- average value:

$$Y_m = \frac{Y}{X}, \qquad (2)$$

- marginal value:
  - in the case of finite variations:

$$Y_M = \frac{\Delta Y}{\Delta X}, \text{ or} \qquad (3)$$

  - in the case of infinitesimal variations:

$$Y_M = \lim_{\Delta X \to 0} \frac{\Delta Y}{\Delta X} = \frac{dY}{dX}, \qquad (4)$$

- variation pace:

$$R_{Y/X} = \frac{\Delta Y/Y}{\Delta X}, \qquad (5)$$

- elasticity:

$$E_{Y/X} = \frac{\Delta Y/Y}{\Delta X/X}, \text{ or } E_{Y/X} = \frac{\Delta Y/\Delta Y}{Y/X} = \frac{Y_M}{Y_m}, \qquad (6)$$

$E_{Y/X}$ $(-1 \le E_{\frac{Y}{X}} \le 1)$ values indicate lower or higher connection intensity.

- DDOI function is a time function; in its turn, this is an economic resource since its saving might generate an increase of productivity and profit. Therefore, time should be included in the DDOI function.
- by calculating $f'_{current} = f'_a$ and $f'_{when\ non-renewable\ resources\ are\ 0} = f'_0$ we can estimate that its value should be close to:

$$f'_{average} = \frac{min\{f'_a, f'_0\} + max\{f'_a, f'_0\}}{2}, \qquad (7)$$

- This is an indicator whose values reflect a hierarchy of the world industrial organizations and which might help to differentiate among various areas of human development [9].

Objectives

- identification of the sustainable development function for industrial organizations, at a general level. For a more specific approach, this function might be further differentiated according to country, region, field of activity.
- identification of the purpose function at the following levels: economic, social, ecological, distribution of mineral, ecological, energy and capital resources; the study of the integration to the ecosphere of newly emerged socio-economic structures.
- identification and calculation of the indicators (parameters) that occur in the expression of this function and the relations among these indicators.
- breaking up into the constituent components according to indicator size.
- identification of sub-component coefficients (influence factors).
- calculating the above and the 3 components of the fDDOI.
- performing marginal calculus.
- calculating the percentage of indicators (regulated or not) in the f'DDOI, f_1, f_2, f_3, according to the formulae:

$$p\% = \frac{x_i}{f'} \; or \; \frac{x_i}{f_1} \; or \; \frac{x_i}{f_2} \; or \; \frac{x_i}{f_3} \quad \Big| (\cdot 10^{-i_k}), \qquad (8)$$

- hierarchy of industrial organizations:
  - according to the sustainable development of branch, county, levels;
  - according to various indicators or values of the f'DDOI, f_1, f_2, f_3.
- open tasks proposal.

Note: The present model has been designed provided the lack of any force majeure circumstances (disasters – earthquakes, fires, pandemia, etc.) when the values of certain indicators may reach zero. Moreover, mention should be made that this model maybe adapted to the situation of economic crisis, when some indicators become null.

3. MATHEMATICAL MODEL

We may define the DDOI function (sustainable development of industrial organizations) meant to assess (measure) sustainable development as [9]:

$$f(t) = \begin{pmatrix} f_1(x_1(t), \dots, x_n(t)), f_2\,(x_{n+1}(t), \dots, x_{n+m}(t)), \\ f_3(x_{n+m+1}(t), \dots, x_{n+m+p}(t) \end{pmatrix} = \left(\frac{\sum_{k=1}^{n} [^{\pm} p_k(t) x_k(t)] 10^{i_k}}{n}, \frac{\sum_{k=1}^{m} [^{\pm} r_k(t) x_{n+k}] 10^{j_k}}{m}, \frac{\sum_{k=1}^{p} [^{\pm} q_k(t)\, x_{n+m+k}(t)] 10^{l_k}}{p} \right), (9)$$

$$f_t = (f_{1t}, f_{2t}, f_{3t}); \;\; f'_t = \frac{f_{1t} + f_{2t} + f_{3t}}{3}, \qquad (10)$$

$$f: \boldsymbol{N}_{2009} \to [0, \infty) \times [0, \infty) \times [0, \infty)$$

$$\boldsymbol{N}_{2009} = \{2009, 2010, 2011 \dots\}$$

$$f': \boldsymbol{N}_{2009} \to [0, \infty)$$

where: t is time expressed in years.

2009 is considered the reference year, since the estimation of non-renewable resources is performed at the beginning of this year.

The vectorial function includes the following components:

$$f_1: \Omega_1 \times \Omega_2 \times \dots \times \Omega_n \to [0,\infty)$$

purpose function at economic level, (11)

$$f_2: \Omega_{n+1} \times \Omega_{n+2} \times \dots \times \Omega_{n+m} \to [0,\infty)$$

purpose funtion at social level, (12)

$$f_3: \Omega_{n+m+1} \times \Omega_{n+m+2} \times \dots \times \Omega_{n+m+p} \to [0,\infty)$$

purpose function at environmental level, (13)

Knowledge of their definition laws (f_1 , f_2 , f_3) entails knowledge of the DDOI f function.

Examples of economic, social and environmental indicators.

3.1. Indicators of the economic component

$$f_1: \Omega_1 \times \Omega_2 \times \dots \times \Omega_n \to [0,\infty)$$

$$f_1(t) = f_1(x_1(t), x_2(t), \dots, x_n(t)) = = \frac{\sum_{k=1}^{n}[{}^{\pm}p_k(t)x_k(t)]10^{i_k}}{n}, \quad (14)$$

$$\boldsymbol{x_1}: \boldsymbol{N_{2009}} \to \Omega_1 \subseteq [\boldsymbol{0},\infty)$$

indicator expressing global liquidity

$$x_1(t) = \frac{Floating\ assests}{urrent\ liabilities} \quad |(t), \quad (15)$$

$\boldsymbol{x_2}: \boldsymbol{N_{2009}} \to \Omega_2 \subseteq [\boldsymbol{0},\infty)$ cost − effectiveness rate

$$x_{12}(t) = \frac{Net\ profit}{Total\ income} 100 \quad |(t), \quad (16)$$

$$\boldsymbol{x_3}: \boldsymbol{N_{2009}} \to \Omega_3 \subseteq [\boldsymbol{0},\infty)$$

indicator expressing financial security

$$x_3(t) = \frac{Company`s\ capital}{Medium-\ and\ long-term\ liabilities} \quad |(t), \quad (17)$$

$$\boldsymbol{x_4}: \boldsymbol{N_{2009}} \to \Omega_4 \subseteq [\boldsymbol{0},\infty)$$

indicator expressing efficiency rat of total expenditure

$$x_4(t) = \frac{Total\ expenditure}{Total\ income} 1000 \quad |(t), \quad (18)$$

$$\boldsymbol{x_5}: \boldsymbol{N_{2009}} \to \Omega_5 \subseteq [\boldsymbol{0},\infty)$$

efficiency of production asset operation

$$x_5(t) = \frac{CA\ or\ Q}{A}, \quad |(t), \quad (19)$$

where: Q is output;

A - expenses depreciation

3.2. Indicators of social component

$$f_2: \Omega_{n+1} \times \Omega_{n+2} \times \dots \times \Omega_{n+p} \to [0,\infty)$$

$$f_2(t) = f_2(x_{n+1}(t), x_{n+2}(t), \dots, x_{n+m}(t)) = \frac{\sum_{k=1}^{m}[{}^{\pm}r_k(t)x_{n+k}]10^{j_k}}{m} \quad (20)$$

Where x_i, $i = \overline{n+1, n+m}$ indicators (parameters) may have the following significances:

$$x_{n+1}: \boldsymbol{N}_{2009} \to \Omega_{n+1} \subseteq [0,\infty)$$

indicator reflecting the average recorded number of staff

$$x_{n+1}(t) = \overline{Ns_t} = \frac{\sum NZ}{\sum Z} \quad |(t), \quad (21)$$

where: N is the number of daily recorded number of employees;

Z - the number of days preserving the respective recorded number of employees.

$$x_{n+2}: \boldsymbol{N}_{2009} \to \Omega_{n+2} \subseteq [0,\infty)$$

indicator reflecting payment of the labour force

$$x_{n+2}(t) = \frac{Total\ salary\ expenditure}{Turnover} \quad |(t), \quad (22)$$

$$x_{n+3}: \boldsymbol{N}_{2009} \to \Omega_{n+3} \subseteq [0,\infty)$$

Indicator of the rate of occupational disease of the total average recorded number

$$x_{n+3}(t) = \frac{Number\ of\ occupational\ disease}{\overline{Ns}} 100 \quad |(t), \quad (23)$$

$$x_{n+4}: \boldsymbol{N}_{2009} \to \Omega_{n+4} \subseteq [0,\infty)$$

Indicator reflecting the rate of bonuses (seniority, difficult/dangerous/ /toxic working conditions) of the total salary amount

$$x_{n+4}(t) = \frac{Total\ accomplished\ bonuses}{total\ amount\ of\ accomplished\ gross\ salary} 100 \ |(t), \quad (24)$$

$$x_{n+5}: \boldsymbol{N_{2009}} \to \Omega_{\boldsymbol{n+5}} \subseteq [0,\infty)$$

organizational wellness indicator [2], [7].

3.3. Indicators of ecological component

$$f_3: \Omega_{n+m+1} \times \Omega_{n+m+2} \times \dots \times \Omega_{n+m+p} \to [0,\infty)$$

$$f_3(t) = f_3\left(x_{n+m+1}(t), x_{n+m+2}(t), \dots, x_{n+m+p}(t)\right) = \frac{\sum_{k=1}^{p}[{}^{\pm}q_k(t)\ x_{n+m+k}(t)]10^{l_k}}{p} \quad (31)$$

$$p_k: N_{2009} \to [0,1], k = \overline{1,p}$$

$$p_k(t) = \frac{a}{10^b}$$

where the x_i, $i = \overline{n+m+1, n+m+p}$ indicators (parameters) may have the following significances [3]:

$\boldsymbol{x_i}: \boldsymbol{N_{2009}} \to \Omega_{\boldsymbol{i}} \subseteq [\boldsymbol{0},\infty)$ indicator of the renewable resources function (eolian energy, solar energy, hydro-energy, biomass, geothermal)

$$i = \overline{n+m+1, n+m+5}$$

$$x_i(t) = 336{,}62|`_{2006} - c\ (t - 2009), \quad t \in \{2009, 2010, \dots\} \quad (32)$$

$x_i(t)$ it represents the amount of existing resources at a given time t;

where: c stands for consumption/year.

$x_{n+m+1}: \boldsymbol{N_{2009}} \to \Omega_{\boldsymbol{n+m+1}} \subseteq [0,\infty)$ it represents the function modelling the amount of petrol

$$x_{n+m+1}(t) = \begin{cases} 145 - 3\,(t - 2009), t \in [2009, 2046] \cap \boldsymbol{N}\ (mld.\,t) \\ \\ 0\ (relative\ zero), t \in \{2047, 2048, \dots\} \end{cases} \quad (33)$$

$x_{n+m+2}: \boldsymbol{N_{2009}} \to \Omega_{\boldsymbol{n+m+2}} \subseteq [0, \infty)$ it represents the function expressing bio-diversity (also called in specialized literature "specific variety").

$$x_{n+m+2}(t) = 1 - \frac{\sum_{i=1}^{S} n_i(t)(n_i(t) - 1)}{N(t)(N(t) - 1)}$$

(indicator of reverse Simpson), (34)

where: $n_i(t)$ it represents the number of individuals from i species ($i = \overline{1, S}$) at a given time t

$x_{n+m+3}: \boldsymbol{N_{2009}} \to \Omega_{\boldsymbol{n+m+3}} \subseteq [0, \infty)$ it represents an indicator expressing significant environmental aspects

$x_{n+m+3}(t) =$ the rate of significant environmental aspects

$x_{n+m+4}: \boldsymbol{N}_{2009} \to \Omega_{n+m+4} \subseteq [0, \infty)$

innovation indicator [6]

$$x_{n+m+4}(t) = F(y_1(t), y_2(t), \dots, y_{19}(t)), \quad (35)$$

4. CHARACTERISTICS

1. The DDOI function is continuous as follows:

If there is a sufficiently small variation ε_i for each indicator (especially if this variation is defined for a single indicator) then the value of fDDOI does not change:

$$\lim_{\varepsilon_i \to 0} f(X - \varepsilon) = f(X) = \lim_{\varepsilon_i \to 0} f(X + \varepsilon), \quad (37)$$

where: $\varepsilon = (\varepsilon_1, \dots, \varepsilon_{m+n+p})$

$X = (x_1, \dots, x_{m+n+p})$

2. Indicators are not essential, however relevant for industrial organizations belonging to the same field
3. $f(X + Y) = f(X) + f(Y)$ additivity characteristic;
4. $f(\propto X) = \propto f(X)$ homogeneity characteristic;
5. fDDOI has an invariant characteristic, that is its expression is not affected by the change of the measurement unit used for indicator expression;
6. There are categories of vectors where fDDOI has the same value, that is:

$$\exists X, Y \text{ și } f(X) = f(Y)\ [1], [4]$$

5. TYPES OF PROPOSED HIERARCHIES

We might identify the place of an industrial organization within its particular branch (within a county) provided we find out a method that enables a comparative approach to industrial organizations and, moreover, to identify its position in the respective community [9].

Model I – here is a Likert scale hierarchy of industrial organization in relation to fDDOI and f'DDOI:

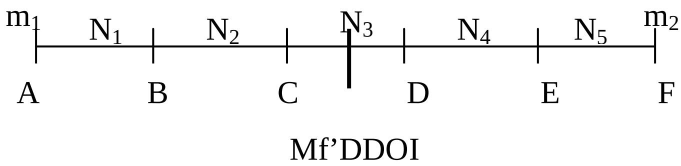
We calculate:

$$M_{f'DDOI} = \frac{\sum f'DDOI}{number\ of\ industrial\ organizations}, \quad (38)$$

this is an average value:

$$m_1 = min\{f'DDOI\} \le M_{f'DDOI} \le max\{f'DDOI\} = m_2,$$

Therefore, we have the following scale:



$$l = \frac{m_2 - m_1}{5},$$

$$A = A(m_1),$$

$$B = B(m_1 + l) = B\left(m_1 + \frac{m_2 - m_1}{5}\right) = B\left(\frac{4m_1 + m_2}{5}\right),$$

$$C = C(m_1 + 2l) = C\left(\frac{5m_1 + 2m_2 - 2m_1}{5}\right) = C\left(\frac{3m_1 + 2m_2}{5}\right),$$

$$D = D(m_1 + 3l) = D\left(\frac{5m_1 + 3m_2 - 3m_1}{5}\right) = D\left(\frac{2m_1 + 3m_2}{5}\right),$$

$$E = E(m_1 + 4l) = E\left(\frac{5m_1 + 4m_2 - 4m_1}{5}\right) = E\left(\frac{m_1 + 4m_2}{5}\right),$$

$$F = F(m_1 + 5l) = F(m_2).$$

The level N_1 includes industrial organizations with the poorest (unsatisfactory) sustainable development or do not observe the limitations of the mathematical model, required by the indicator values.

The level N_2 includes industrial organizations with satisfactory sustainable development.

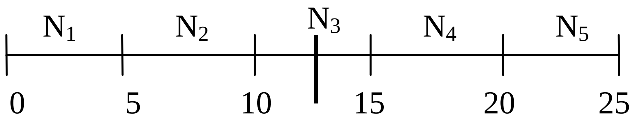
The level N_3 includes industrial organizations with average sustainable development.

The level N_4 includes industrial organizations with good sustainable development.

The level N_5 includes industrial organizations with very good sustainable development.

Model II – hierarchy proposal

Since there is no measurement of sustainable development for similar types of organizations, their hierarchy has not been performed so far; therefore we suggest the following:



We have agreed upon this scale as a result of the analyses performed in the case study from [9].

Model III – a hierarchy of industrial organizations including fDDOI as a criterion

We are going to use the $A=(a_{ij})_{\substack{i=\overline{1,m}\\ j=\overline{1,n}}}$ matrix of the „accomplishments" of specific industrial organizations. Some of these criteria C_j are the f'DDOI; f_1,f_2,f_3 values.

OI / Criteria	$OI_1 \ldots OI_i \ldots OI_n$	
C_1	$a_{11} \ldots a_{1i} \ldots a_{nm}$	$\propto_1$
⋮	⋮ ... ⋮ ... ⋮	⋮
⋮	⋮ ... ⋮ ... ⋮	⋮
C_j	⋮ ... a_{ij} ... ⋮	$\propto_j$
⋮	⋮ ... ⋮ ... ⋮	⋮
⋮	⋮ ... ⋮ ... ⋮	⋮
C_n	$a_{n1} \ldots a_{ni} \ldots a_{nm}$	$\propto_n$
	$\beta_1 \ldots \beta_i \ldots \beta_m$	

Step 1. Calculating $\propto_j = \max_{i=\overline{1,m}} a_{ij}$ for each criterion $C_j,\ j=\overline{1,n}$.

Step 2. Calculating $\beta_j = \min_{i=\overline{1,n}} a_{ij}$ pentru fiecare $OI_i,\ i=\overline{1,m}$.

Step 3. Calculating $\propto_{j0} = \min_{j=\overline{1,n}} \propto_j = \min_{j=\overline{1,n}} \max_{i=\overline{1,m}} a_{ij}$ and $\beta_{i0} = \max_{i=\overline{1,m}} \beta_i = \max_{i=\overline{1,m}} \min_{j=\overline{1,n}} a_{ij}$.

Step 4. Calculating $\gamma_{ij} = |a_{ij} - \propto_{j0}|,\ i=\overline{1,m},\ j=\overline{1,n};\ \Gamma = (\gamma_{ij})_{\substack{i=\overline{1,m}\\ j=\overline{1,n}}}$.

Step 5. Calculating $\gamma'_{ij} = |a_{ij} - \beta_{i0}|,\ i=\overline{1,m},\ j=\overline{1,n};\ \Gamma' = (\gamma'_{ij})_{\substack{i=\overline{1,m}\\ j=\overline{1,n}}}$.

The hierarchy of industrial organizations is performed top-down according to $\gamma_{ij},\ i=\overline{1,m}$ values of the Γ matrix. That is: if $\gamma_{i_1 j} \geq \gamma_{i_2 j}$ then $OI_{i_1} \leq OI_{i_2}$ (OI_{i_1} has a poorer sustainable development than OI_{i_2}).

Within the same organization, the hierarchy of an optimal accomplishment of the activity, in keeping with $C_1, \ldots, C_n$ criteria may be thus interpreted: if $\gamma'_{ij_1} \geq \gamma'_{ij_2}$, then OI_i performed a more efficient activity according to C_{j_2} criterion rather than C_{j_1}.

Comments

A special case is $\propto_{j0} = \beta_{i0}$, when the sustainable development of OI_{i_0} is optimal compared to other industrial organizations, as well as in keeping with all assessment criteria.

Criteria C_j are indicators x_i or f' or f_1, f_2, f_3.

The algorithm can be programmed (for instance: C and C++).

6. CONCLUSIONS AND OPEN TASKS

1. Performing comparative analyses of f'; f_1, f_2, f_3 values and the values of various indicators (for instance in SPSS) and the interpretation of results by means of Excel programs.

2. Accomplishing optimization problems for fDDOI components; f_1, f_2, f_3 or f'DDOI, or f'DDOI with practical applicability, that might have been solved by means of simplex algorithms or other algorithms related to operational research.

3. The approach to the mathematical model resorting to factorial and multifactorial analyses. Factorial analysis was designed by Charles Spearman as a research method consisting in the representation of a series of n variables, all characterizing the same population, by means of a limited number of independent factors (characteristics). These characteristics may be economic, social and environmental indicators.

After calculating the correlation coefficients among these characteristics, taken two by two, one can note that there are more or less tight statistical relations. There are two aspects related to factorial analysis:

a) checking certain statistical hypotheses regarding the number of factors

b) estimating the parameters occurring in the expression of characteristics

$$Z_i(Z_i = a_i F + e_i, \qquad i=\overline{1,n})$$

where: a_i = model parameters, F = common factor

or hypothetical variant, e_i = specific factors.

The quantities Z_i are noticeable, whereas F, e_i și a_i are not noticeable. The latter may be inferred however by solving the mathematical model.

When there is a linear connection among variables, as a cloudy elongated shape of points (fig. 1), surrounding a straight line, then we have a single common denominator, which is geometrically limited to a linear multitude, that is to the points of the straight line.

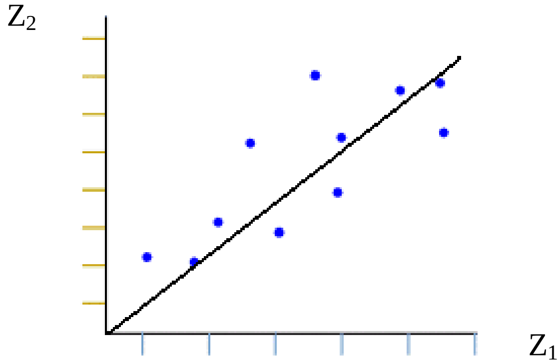


Fig.1 Linear dependence

If we refer to three variables, Z_1, Z_2 și Z_3, for which the measured values are represented in the three dimensional area by means of a flat cloudy shape of points, we have the situation of a two-factor representation, that is the axes of the flat shaped drawing that regulate the respective cloud.

According to the Spearman criterion, the correlation matrix:

$$F = \begin{pmatrix} 1 & r_{12} & r_{13} \cdots \cdots r_{1n} \\ r_{21} & 1 & r_{23} \cdots \cdots r_{2n} \\ \cdots & \cdots & \cdots\cdots\cdots\cdots\cdots\cdots \\ r_{n1} & r_{n2} & r_{n3} \cdots \cdots 1 \end{pmatrix}$$

accepts a single common denominator provided that its tetradic variables are annuled.

$$r_{ab}r_{cd} - r_{bc}r_{cd};\;\; a,b,c,d = \overline{1,n};\;\; a \neq b \neq c \neq d.$$

Practically, this condition is rarely verified, therefore it is considered fulfilled when these differences are close to zero.

An approach to the global economic crisis, the sustainable development of industrial organizations undergoing such a situation, by means of *chaos theory*.

Procedure proposal for its implementation and model application shall be made as accurately as possible, at the level of each industrial organization, branch of field of activity.

Creating sustainable development departments at the level of each industrial organization that should establish the values for the influence factors for the indicators occurring in the fDDOI expression, in relation to the specificity and activity of the industrial organization they belong to.

Identification at the country level of certain value intervals for the influence factors, for the industrial organizations that belong to the same branch.

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